A Non-structuralist Alternative to Formalism

In the *Critique of Pure Reason* Kant distinguishes two uses of reason: an intuitive use that he held was the use to which reason is put in mathematical practice, and a discursive use that is employed in philosophical work. The intuitive use, on Kant's account, involves the construction of concepts in pure intuition and enables demonstrations that at once extend our knowledge (are ampliative) and are a priori, that is, necessary, though not logically necessary. The discursive use instead involves reasoning directly from concepts and cannot in the same way extend our knowledge. Not synthetic judgments a priori but only principles (that are at once synthetic and a priori, but yet not judgments) are the fruits of the discursive use of reason in philosophy.

Over the course of the nineteenth century, developments in mathematics seemed definitively to show that Kant was wrong. Although in Kant's day mathematicians did make extensive use of paper-and-pencil manipulations— whether those involving Euclidean diagrams or those involving instead the symbolic language of mathematics first introduced in the seventeenth century and greatly expanded in the eighteenth—in the nineteenth century mathematicians began to reason directly on the basis of concepts. As Dedekind explains (1877):

a theory based upon calculation [that is, paper-and-pencil manipulations] would, as it seems to me, not offer the highest degree of perfection; it is preferable, as in the modern theory of functions, to seek to draw the demonstrations, no longer from calculations, but directly from the characteristic fundamental concepts, and to construct the theory in such a way that it will, on the contrary, be in a position to predict the results of the calculation.

Not an intuitive use of reason but instead a discursive use, a use of reason that was deductive and directly from concepts, was to be the way forward in mathematics.

But although this development in the practice of mathematics over the course of the nineteenth century was generally taken to show that Kant was wrong to think that mathematics involves an intuitive use of reason, that is, ampliative constructions, there were two notable exceptions: Frege and, independently, Peirce. Frege claims in his 1884 *Foundations of Arithmetic*, section 88, that deductive reasoning on the basis of concepts can be at once deductive (that is, analytic in Kant's sense) and also ampliative (hence, synthetic). And Peirce claims similarly, in "The Logic of Mathematics in Relation to Education" (1898), that "Kant . . . fell into error in supposing that mathematical and philosophical necessary reasoning are distinguished by the circumstance that the former uses constructions. This is not true. All necessary reasoning whatsoever proceeds by constructions." Both suggest, then, that nineteenth-century developments in mathematical practice show not that Kant was wrong to think that reasoning in mathematics is constructive rather than deductive from concepts but that he was wrong about the nature of deductive reasoning. This idea, that even deductive reason can be also constructive and ampliative, provides a way forward that is interestingly different from both formalism and structuralism.

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