Klein's invariant-theoretic structuralism

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An important development in nineteenth-century mathematics towards a "structuralist" conception of the discipline is related to work on invariants. Classical invariant theory was established as an algebraic research field in the second half of the nineteenth century in work by A. Cayley, J. Sylvester, and D.Hilbert as the study of polynomial functions that remain invariant under transformations from given linear group. In the context of geometry, invariants became of central importance in work by Felix Klein, in particular, in his well-known Erlangen program. Klein's programmatic article *Vergleichende Betrachtungen über neuere geometrische Forschungen* (1872) lays out a new methodology for geometrical research. Roughly put, the central idea is to classify geometries group theoretically, that is in terms of the properties of spatial objects that remain stable under a given group of transformations. Given this approach, different types of spaces (i.e. affine, Euclidean, projective space, etc.) can each be characterized in terms of their invariant properties. Moreover, given that the transformation groups corresponding to such spaces are typically related by group inclusion, Klein saw that the geometrical theories describing them can be ordered and classified in terms of their corresponding groups.

In the talk, I will give a closer discussion of Klein's group-theoretical approach in geometry and analyze its structuralist underpinnings. It has often been stated that the Erlanger Program has contributed significantly to a "structural turn" in modern mathematics. But what precisely is the nature of the structuralism underlying this research program? To address this, the talk will focus on a central conceptual assumption underlying Klein's proposal to redefine geometry as a form of invariant theory: geometry is no longer conceived here as the study of particular figures in space but rather as the study of the properties of such figures that remain invariant under structure-preserving transformations. Given this account, we can say that the subject matter of a given geometry is fully specified by its corresponding group of transformations and thus by the abstract structure described by this group. Moreover, as Klein showed in his work on transfer principles, two geometries can describe very different spatial objects but nevertheless be structurally equivalent if their corresponding transformation groups are isomorphic. This is, as I will argue in the talk, clearly a structuralist approach, similar in several respects to modern thinking about mathematics in category-theoretic terms and to categorical structuralism more specifically.